Hong Kong Mathematics Olympiad (1994 – 95)

Final Event 6 (Group)

香港数学竞赛 (1994 – 95)

决赛项目 6 (团体)

$2^a \cdot 9^b$ is a four digit number and its thousands digit is 2, its hundreds digit is a, its tens digit is 9 and its units digit is b, find a, b.

$2^a \cdot 9^b$ 为一四位数，其千位数是 2，百位数是 a，十位数是 9，个位数是 b，求 a 及 b。

$a =$

$b =$

Find c, if $c=\left(1+\frac{1}{2}+\frac{1}{3}\right)\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)-\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)\left(\frac{1}{2}+\frac{1}{3}\right)$.

若 $c=\left(1+\frac{1}{2}+\frac{1}{3}\right)\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)-\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)\left(\frac{1}{2}+\frac{1}{3}\right)$，求 c。

$c =$

Find d, if

$$d=\left(1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{1994}\right)\left(\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{1995}\right)-\left(1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{1995}\right)\left(\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{1994}\right).$$

求 d，若

$$d=\left(1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{1994}\right)\left(\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{1995}\right)-\left(1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{1995}\right)\left(\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{1994}\right)。$$

$d =$

Hong Kong Mathematics Olympiad (1994 – 95)

Final Event 7 (Group)

香港数学竞赛 (1994 – 95)

决赛项目 7 (团体)

(i) Let p, q, r be the three sides of triangle PQR. If

$$p^4+q^4+r^4=2r^2\left(p^2+q^2\right)$$

find a, where $a=\cos^2 R$ and R denotes the angle opposite r.

设 p、q、r 为三角形 PQR 的三边。若

$$p^4+q^4+r^4=2r^2\left(p^2+q^2\right)$$

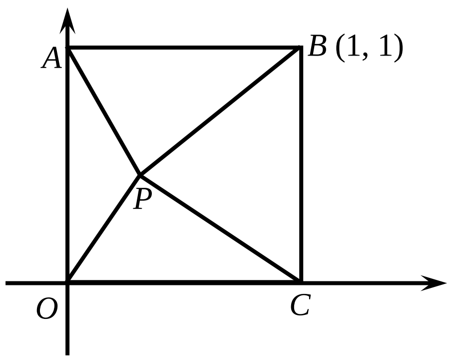
且 $a=\cos^2 R$ 其中 R 的对边为 r，求 a。

$a=$

(ii) Refer to the diagram, P is any point inside the square OABC and b is the minimum value of $PO+PA+PB+PC$, find b.

如图，P 为正方形 $OABC$ 内的任意点，且 b 为 $PO+PA+PB+PC$ 之最小值，求 b。

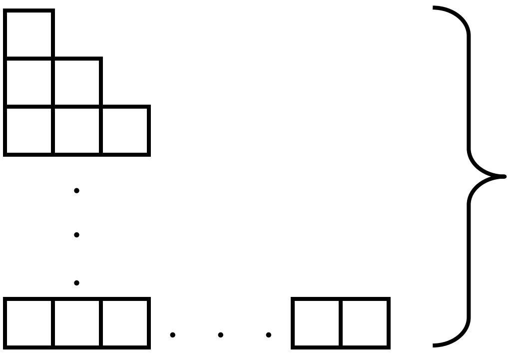
$b=$



(iii) Identical matches of length 1 are used to arrange the following pattern, if c denotes the total length of matches used, find c.

长度同为 1 的火柴被排成下列图案。若以 c 表示用去火柴枝的总长，求 c。

$c =$



(iv) Find d, where $d = \sqrt{111111 - 222}$.

求 d，若 $d = \sqrt{111111 - 222}$。

$d =$

Hong Kong Mathematics Olympiad (1994 – 95)

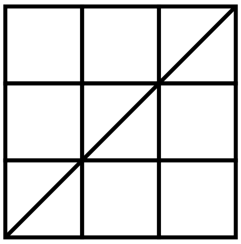
Final Event 8 (Group)

香港数学竞赛 (1994 – 95)

决赛项目 8 (团体)

Rectangles of length ℓ and breadth b where ℓ, b are positive integers, are drawn on square grid paper. For each of these rectangles, a diagonal is drawn and the number of vertices V intersected (excluding the two end points) is counted (see figure) .

在方格纸上绘画尺寸为 $\ell \times b$ 的长方形，其中 ℓ、b 为正整数并添上对角线一条。以 V 代表相交的端点总数 (不包括首尾两点在内) 。 (如图示)



$\ell = b = 3$

$V = 2$

(i) Find V, when $\ell = 6$, $b = 4$.

当 $\ell = 6$， $b = 4$ 时，求 V。

$V =$

(ii) Find V, when $\ell = 5$, $b = 3$.

当 $\ell = 5$， $b = 3$ 时，求 V。

$V =$

(iii) When $\ell = 12$ and $1 < b < 12$, find r, the number of different values of b that makes $V = 0$?

当 $\ell = 12$ 且 $1 < b < 12$ 时，求使 $V = 0$ 时，b 的不同个数 r 。

$r =$

(iv) Find V, when $\ell = 108$, $b = 72$.

当 $\ell = 108$， $b = 72$ 时，求 V。

$V =$

Hong Kong Mathematics Olympiad (1994 – 95)

Final Event 9 (Group)

香港数学竞赛 (1994 – 95)

决赛项目 9 (团体)

A , B , C , D are different integers ranging from 0 to 9 and

$$\begin{array}{rcccc} & A & A & B & C \\ - & B & A & C & B \\ \hline & D & A & C & D \end{array}$$

Find A , B , C and D .

A 、 B 、 C 、 D 为由 0 至 9 间的不同整数，且

$$\begin{array}{rcccc} & A & A & B & C \\ - & B & A & C & B \\ \hline & D & A & C & D \end{array}$$

求 A 、 B 、 C 及 D 之值。

A =	B =	C =	D =

Hong Kong Mathematics Olympiad (1994 – 95)
Final Event 10 (Group)
香港数学竞赛 (1994 – 95)
决赛项目 10 (团体)

Lattice points are points on a rectangular coordinate plane having both x- and y-coordinates being integers. A moving point P is initially located at $(0, 0)$. It moves 1 unit along the coordinate lines (in either directions) in a single step.

在直角坐标平面上，x- 和 y- 坐标同为整数的点称为格点。P 是起始时位于 $(0, 0)$ 的移动点，它每一步必须沿坐标线的其中一过个方向走 1 个单位的距离。

(i) If P moves 1 step then P can reach a different lattice points, find a.

若 P 走 1 步，它可到达 a 个格点，求 a。

$a =$

(ii) If P moves not more than 2 steps then P can reach b different lattice points, find b.

若 P 可走不超过 2 步，它可到达 b 个格点，求 b。

$b =$

(iii) If P moves 3 steps then P can reach c different lattice points, find c.

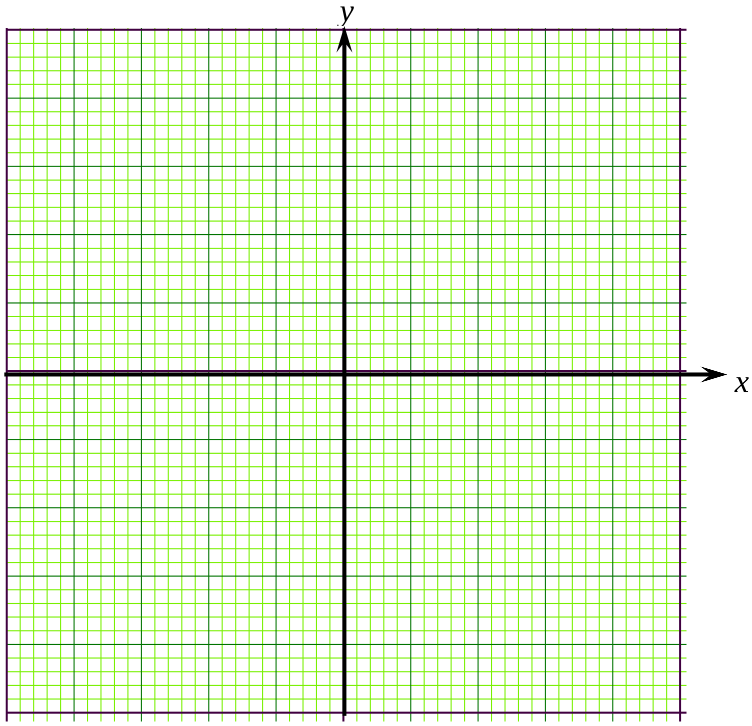
若 P 走 3 步，它可到达 c 个格点，求 c。

$c =$

(iv) If d is the probability that P lies on the straight line $x + y = 9$ when P advances 9 steps, find d.

若 P 走 9 步，它停在直线 $x + y = 9$ 上的概率是 d，求 d。

$d =$


y
x